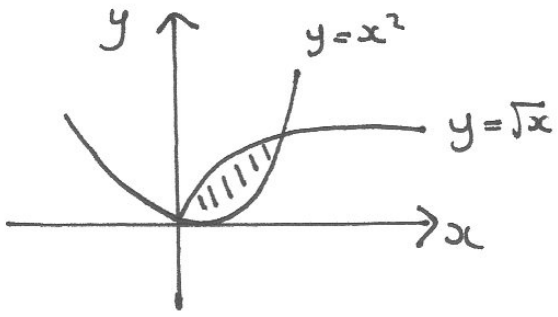
Exercise 7 page 137

Q6. $y = x^2$, $y = \sqrt{x}$



To find the limits – equate

so $x^2 = \sqrt{x}$ ← square both sides to get rid of the $\sqrt{}$

$\Rightarrow x^4 = x$

$\Rightarrow x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0 \quad x^3 - 1 = 0$

$x^3 = 1 \Rightarrow x = 1$

so limits $\underline{x = 0 \quad x = 1}$

Area between curves $= \int_0^1$ upper curve – lower curve dx

$$= \int_0^1 \sqrt{x} - x^2 \, dx$$

$$= \int_0^1 x^{1/2} - x^2 \, dx$$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_0^1 = \left[\frac{2}{3}\sqrt{x}^3 - \frac{x^3}{3} \right]_0^1$$

$$= \left(\frac{2}{3}\sqrt{1}^3 - \frac{1^3}{3} \right) - \left(\frac{2}{3}\sqrt{0}^3 - \frac{0^3}{3} \right)$$

$$= \left(\frac{2}{3} - \frac{1}{3} \right) - 0 = \underline{\underline{\frac{1}{3}}}$$

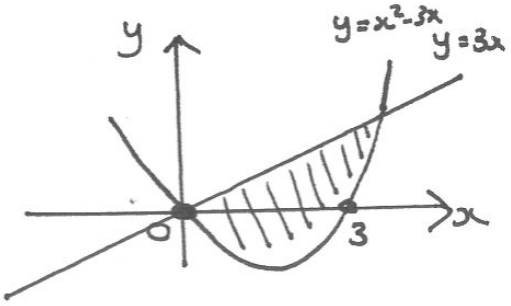
Q7 $y = x^2 - 3x$ $y = 3x$ ← straight line through (0,0)

↓ find roots to draw parabola

let $y=0$ $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \quad x = 3$



Find limits

$\left.\begin{matrix} y = x^2 - 3x \\ y = 3x \end{matrix}\right\}$ equate

$x^2 - 3x = 3x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0 \quad x = 6$

$$\text{Area} = \int_0^6 \text{upper} - \text{lower}\ dx$$

$$= \int_0^6 3x - (x^2 - 3x)\ dx$$

$$= \int_0^6 3x - x^2 + 3x\ dx = \int_0^6 6x - x^2\ dx$$

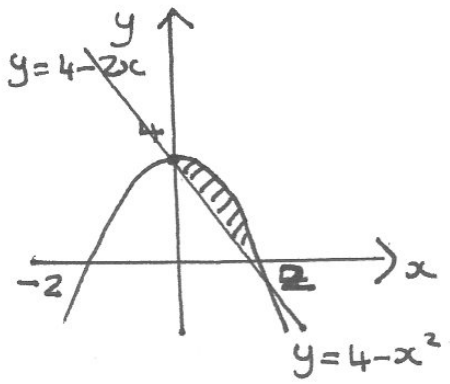
$$= \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]_0^6 = \left[3x^2 - \frac{x^3}{3}\right]_0^6$$

$$= \left(3(6)^2 - \frac{6^3}{3}\right) - (0)$$

$$= 108 - \frac{216}{3} = 108 - 72 = \underline{\underline{36}}$$

Q8 $y = 4 - x^2$

$y = 4 - 2x$



Limits

$4 - x^2 = 4 - 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad x = 2$

$$\text{Area} = \int_0^2 (4 - x^2) - (4 - 2x)\, dx$$

$$= \int_0^2 4 - x^2 - 4 + 2x\, dx$$

$$= \int_0^2 2x - x^2\, dx$$

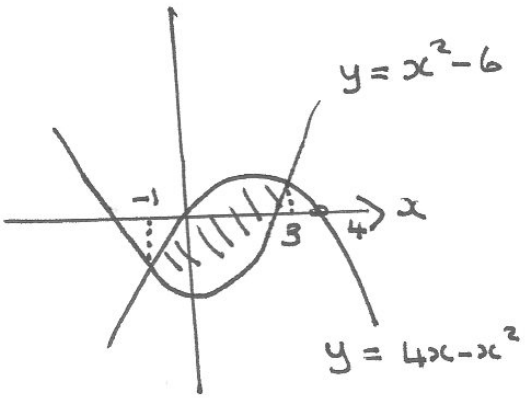
$$= \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$= \left(2^2 - \frac{2^3}{3}\right) - (0)$$

$$= 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \underline{\underline{\frac{4}{3}}}$$

10. $y = 4x - x^2$

$y = x^2 - 6$



Find limit – equate

$4x - x^2 = x^2 - 6$

$2x^2 - 4x - 6 = 0$

$2(x^2 - 2x - 3) = 0$

$2(x+1)(x-3) = 0$

$x = -1 \quad x = 3$

$$\text{Area} = \int_{-1}^{3} (4x - x^2) - (x^2 - 6)\, dx$$

$$= \int_{-1}^{3} 4x - x^2 - x^2 + 6\, dx$$

$$= \int_{-1}^{3} 4x - 2x^2 + 6\, dx$$

$$= \left[2x^2 - \frac{2x^3}{3} + 6x \right]_{-1}^{3}$$

$$= \left(2(3)^2 - \frac{2}{3}(3)^3 + 3(3)^2 \right) - \left(2(-1)^2 - \frac{2}{3}(-1)^3 + 3(-1)^2 \right)$$

$$= (18 - 18 + 27) - \left(2 + \frac{2}{3} + 3\right)$$

$$= (27) - 2 - \frac{2}{3} - 3$$

$$= 22 - \frac{2}{3} = 21\frac{1}{3}$$